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DETERMINATION OF ELASTIC CONSTANTS OF ORTHOTROPIC MATERIALS WITH SPECIAL REFERENCE TO LAMINATES

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88

Determination of Elastic Constants of Orthotropic Materials with Special Reference to Laminates*

By R. K. Witt, W. H. Hoppmann, II, and R. S. Buxbaum[1]

SYNOPSIS

In this paper there are given the theoretical basis and an experimental method for determining the anisotropic elastic constants of a material by measuring deflections of a thin plate of the material subject to couples on its boundary. Using the method, elastic constants are determined for three different laminates.

In recent years there has been a great interest in materials that have anisotropic elastic properties. These materials include in particular glass-fabric laminates.

For technological purposes of design, for example vibration considerations, it is necessary to know the various compliance or stiffness factors of a material as presented by its elastic constants. Previous papers (**1,2,3**)[2] have presented various methods for determining these constants. In the present work some of these methods have been adapted to the purpose of determining the elastic constants from measurements of the deflections of thin plates loaded on the boundary by bending and twisting couples. Three laminates with widely different properties have been tested.

NOMENCLATURE

ϵ_x = extensional strain in x direction,
ϵ_y = extensional strain in y direction,
γ_{xy} = shear strain in $x-y$ plane,
σ_x = extensional stress in x direction,
σ_y = extensional stress in y direction,
τ_{xy} = shear stress in $x-y$ plane,
S_{ij} = elastic constants ($S_{ij} = S_{ji}$),
E_x = $\frac{1}{S_{11}}$ = Young's modulus,
E_y = $\frac{1}{S_{22}}$ = Young's modulus,
ν_{xy} = $-E_x \cdot S_{12}$ = Poisson's ratio,
ν_{yx} = $-E_y \cdot S_{12}$ = Poisson's ratio,
G_{xy} = $\frac{1}{S_{66}}$ = rigidity or shear modulus,
A, B, C = constants of integration,
W = deflection of plate,
h = thickness of plate,
a = width of plate (bending),
b = length of plate (bending),
c = length of side of twister,
M_B = bending couple per unit length distributed uniformly on two opposite edges of plate,
x, y = coordinates of point on plate,
M_T = twisting couple per unit length of side of plate, and
A_x, A_{xy}, A_y = constants in differential equation of plate and are expressions in terms of the elastic constants.

THEORY

The general theory of anisotropic elasticity is developed in the technical literature (**4,5**). In this paper the material investigated is essentially orthotropic and therefore has three axes of symmetry. The test plates are rectangular in shape and cut with their edges parallel to these axes of elastic symmetry. Therefore in the usual notation (**6**) the stress-strain relations may be written for the strains in terms of the stresses and elastic constants as follows:

$$\epsilon_x = S_{11}\sigma_x + S_{12}\sigma_y = \frac{\sigma_x}{E_x} - \frac{\nu_{xy}}{E_x}\sigma_y$$

$$\epsilon_y = S_{21}\sigma_x + S_{22}\sigma_y = -\frac{\nu_{yx}}{E_y}\sigma_x + \frac{\sigma_y}{E_y}$$

$$\gamma_{xy} = S_{66}\tau_{xy} = \frac{\tau_{xy}}{G_{xy}}$$

There are then essentially four constants of elasticity: S_{ij} or G_{xy}, E_x, E_y, and

$$\frac{\nu_{xy}}{E_x} = \frac{\nu_{yx}}{E_y}$$

As may be readily shown, the displacement caused by uniformly distributed bending couples (1) M_B on two opposite sides or twisting couples M_T on all sides, is as follows. For bending, we have:

$$W = \frac{6M_B}{h^3}(S_{11}x^2 + S_{ij}y^2) + Ax + By + C \quad \ldots\ldots(1)$$

$i = 1, 2$ corresponding to $j = 1, 2$ appropriate elastic constant.

For twisting, we have:

$$W = \frac{6M_T}{h^3} \cdot S_{66} \cdot xy + Ax + By + C \quad \ldots(2)$$

The equation of equilibrium for the bending of thin orthotropic plates loaded only with edge couples (**6**) is:

$$A_x \frac{\partial^4 W}{\partial x^4} + A_{xy}\frac{\partial^4 W}{\partial x^2 \partial y^2} + A_y \frac{\partial^4 W}{\partial y^4} = 0 \quad \ldots\ldots(3)$$

Since Eq 3 is of the fourth order it is readily seen that Eqs 1 and 2 which are quadratic in x and y satisfy it. Also it can easily be shown that the boundary conditions are satisfied and that Eqs 1 and 2 are really solutions for the type of loading used in this investigation. The constants A, B, and C in Eqs 1 and 2 are determined from the condition that the plates are supported at three points in the case of bending and symmetrically at two diagonally opposite points in twisting.

The determination of the elastic constants on the basis of these results is illustrated later by a numerical calculation. It is here obvious that one may measure W, M_B, h, x, and y for Eq 1 for the two principal directions of the plates and obtain two simultaneous equations for the determination of the elastic constants S_{ij}. It is also clear that one may measure W, M_T, h, x, and y for Eq 2 and readily determine S_{66} which gives the shearing modulus G_{xy}.

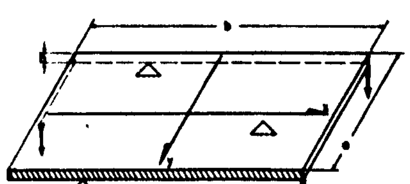

Fig. 1(a).—Bending Plate.

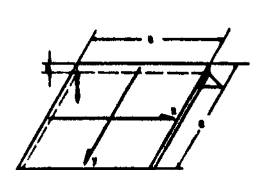

Fig. 1(b).—Twisting Plate.

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[2] The boldface numbers in parentheses refer to the list of references appended to this paper

Experimental Method

From the theoretical formulation it is clear what measurements must be made to determine the elastic constants. For the case of bending it is sufficient to measure the deflections at the origin of coordinates and a few inches off center along the y-axis shown in Fig. 1. For the determination of the shearing modulus, G_{xy}, it is clear that only a single deflection is required and this is measured at the origin of coordinates. Unlike the methods used by others, as previously mentioned, the deflections in this investigation were referred to an absolute rigid frame supported independent of the test plate as shown in Figs. 2 and 3. A rigid pipe flange of 22 in. inside diameter was machined to take a rotatable ground bar on which a sliding steel block is mounted. To this block the deflection gage or micrometer is attached. The pipe flange is supported on top of three equiheight columns of sufficient stiffness. The rotatable bar can be rotated around the center of the pipe flange and the block holding the micrometer fixed can be slid axialwise along the smooth bar. In this fashion the deflection at any point can be measured.

For nonconducting materials, it is necessary to provide an indicator to show precisely when the micrometer touches the plate; otherwise the operator might unconsciously bore the point of the micrometer into the material and cause deflections of the plate by unknown pressures from the micrometer. For this purpose a thin disk of aluminum foil about $\frac{1}{8}$ in. in diameter and about 0.001 in. thick was cemented to the surface of the plate at any location at which deflection was to be measured. A fine electric wire was soldered to this disk and connected to a pilot relay indicator which has a magnetic relay whose clicking is very helpful in quickly showing that contact has been made even under extremely small pressure. The setup is shown in Figs. 2 and 3.

For the purpose of loading the plates, a lever was usually used as shown in Fig. 2. This lever provides a known load on an auxiliary bar or plate suspended from the test plate. The auxiliary plate or bar in the bending test provides forces on two opposite ends of the test plate, and within the region between supports on the test plate essentially uniform moments are produced across the width of the plate in the vicinity of points where deflections are measured. In the case of the twisting test an auxiliary bar was suspended from two opposite corners of the test plate and loaded at its center by a lever or by small weights applied directly in the cases of small required loads.

The load was gradually increased and the corresponding deflection measured at the chosen point on the plate. The results give a load deflection curve for that particular point. For a linear relation it is the slope of this line that gives a deflection per unit load, $[P/W]$, for use with Eqs 1 and 2 in determining elastic constants.

In order to check the method and the accuracy of the test a calibration test was made with medium steel of known elastic constants. The moduli for the steel plates as determined with this test apparatus were 34×10^6 psi for Young's modulus, 12×10^6 psi for the shear modulus and 0.28 for Poisson's ratio. A measured Young's modulus in a standard tension testing machine for the same steel was 32×10^6 psi. It was considered that this constituted a reasonable check of the test procedure and method.

The materials and thicknesses of the 5.25-in. square test sections were as follows:

Glass Base Melamine Resin (GMG)	0.130 in.
Paper Base Type PBG (XX)	0.127 in.
Glass Base Silicone Resin (GSG)	0.116 in.

The over-all length of bending specimen was 10.5 in.

Two bending specimens were used for each material. One was cut so that the length of the rectangular plate was parallel to one principal direction, and the other was cut so that its length was parallel to the other principal direction. These two material-property directions are at right angles to each other in the material.

Only one twisting specimen is required. It was cut from the material so that its sides were respectively parallel to to the principal axes of the material.

For the twisting tests, measurements of deflection were made for both sides of the plate. After the test on one side, the plate was turned over and measurements made on the other side. Measurements for both sides of plates in the bending tests were made only for the

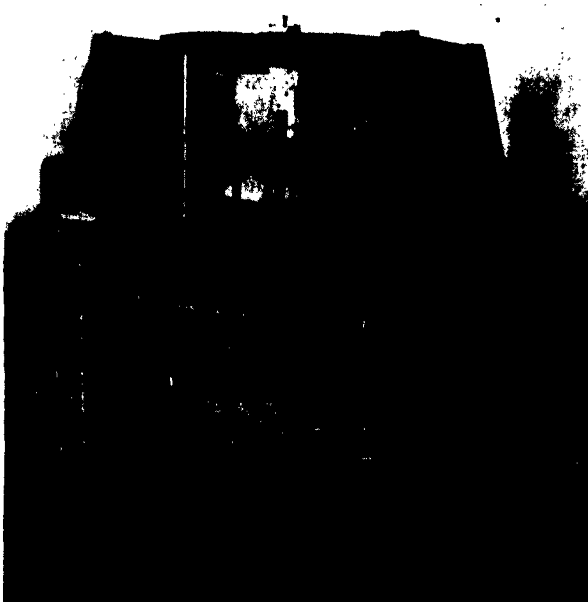

Fig. 2.—Twisting Test.

Fig. 3.—Bending Test.

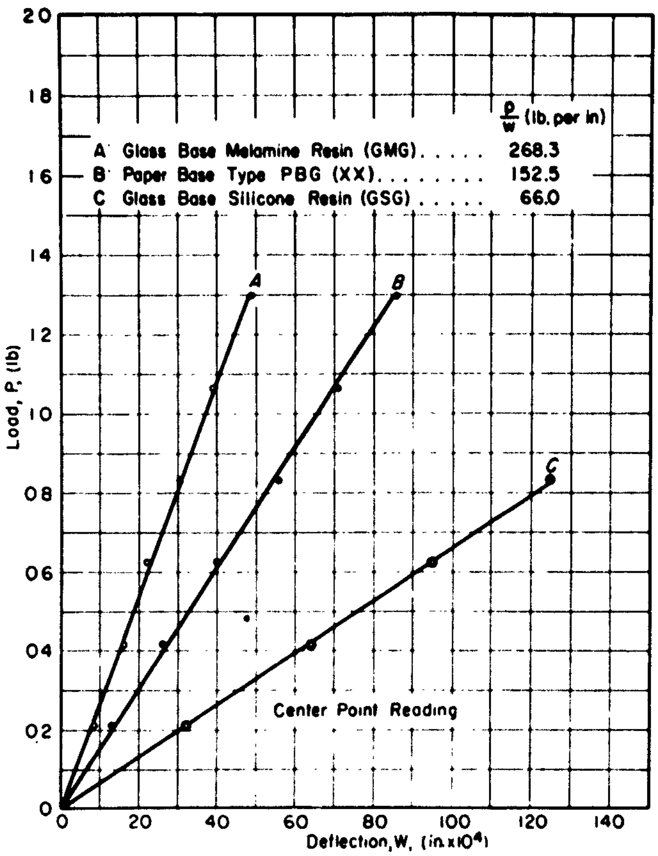

Fig. 4.—Twisting Test.

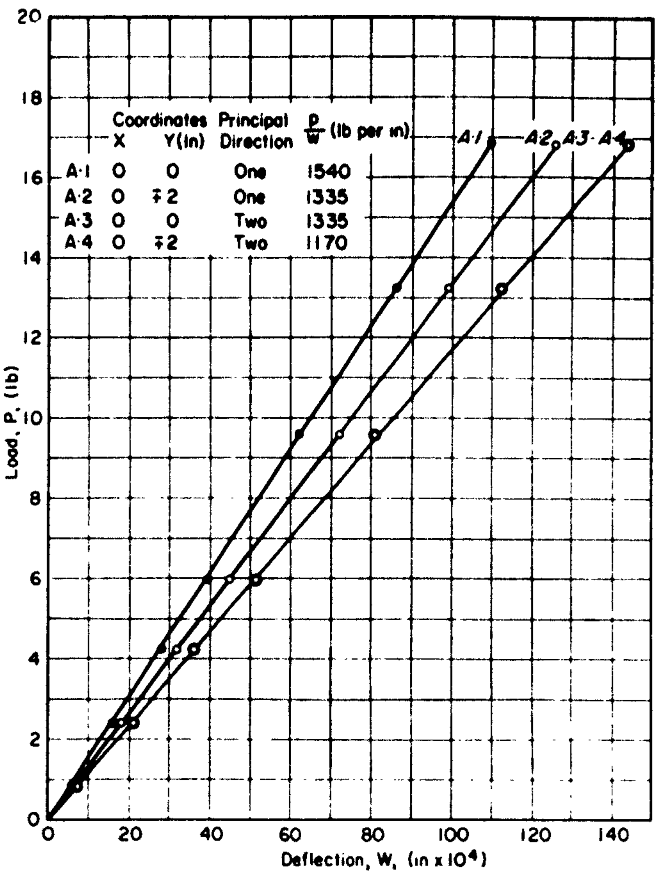

Fig. 5.—Bending Test of Glass Base Melamine Resin (GMG).

glass-base melamine resin. In the bending test, deflections were measured on the y-axis both above and below the x-axis. The measurements were averaged.

Results of Experiments

The force-deflection curves, from which are derived the moment-deflection relations, are plotted in Figs. 4 to 7. These represent twisting data for one plate each of the three materials and bending data for two directions each of the three materials. The slopes of these curves provide the $[P/W]$ values needed in calculating the elastic constants.

Only small percentage differences were observed in the load deflection data as taken for the two sides of the plate.

The bending deflections are for the center point of the plate and for points 2 in. above and 2 in. below the x-axis along the y-axis. Both of these measurements above and below the x-axis are not required for determination of constants, but it is considered that their average is a better result. When deflections are taken for both sides four such readings are averaged, two for each side of the plate.

Numerical Calculation

As an example, the numerical calculations are presented for the glass-base silicone resin. The shear modulus will be calculated first. As given in Eq 2 we have:

$$W = \frac{6M_T}{h^3} \cdot S_{66} \cdot xy + Ax + By + C$$

Now it is well known (6) that the twisting couples for this case can readily be written in terms of the reactions at the supports as follows:

$$M_T = \frac{P}{4}$$

so that we may write:

$$W = \frac{3}{2}\frac{P}{h^3} \cdot \frac{xy}{G_{xy}} + Ax + By + C$$

Since the supports are at the same height and $W = 0$ at supports, we can determine A, B, and C and find that

$$W = \frac{6M_T}{h^3} \cdot \frac{1}{G_{xy}} \cdot (xy + 6.25)$$

At $x = 0, y = 0$

$$W = +\frac{6M_T}{h^3} \cdot \frac{1}{G_{xy}} \times 6.25$$

Hence

$$G_{xy} = \frac{3 \times 6.25}{2h^3}\left[\frac{P}{W}\right]$$

From Fig. 4, curve C $[P/W]$ is 66 lb per in. The plate thickness is 0.116 in. Hence

$$G_{xy} = \frac{3 \times 6.25}{2(0.116)^3} \times 66 = 398{,}000 \text{ psi}$$

Now to calculate bending moduli and Poisson type ratios use Eq 1. We have:

$$W = \frac{6M_B}{h^3}(S_{11}x^2 + S_{12}y^2) + Ax + By + C$$

The plate is supported on three equiheight columns and the coordinates of the support points are (2.5, 0), (−2.5, 2.5), and (−2.5, −2.5).

$$M_B \cdot a = \frac{P}{2} \times 2 \text{ in.} \cdot 5$$

a = width of plate, and M = bending moment per unit width.

Then we have:

$$-ah^3\left[\frac{W_{x,y}}{P}\right] = S_{11}(7.5x^2 - 46.874) + S_{12}(7.5y^2 + 9.375x - 23.4375)$$

For principal direction No. 1, Fig. 7, at (0, 0), $\left[\frac{P}{W}\right]$ = 720 lb per in.

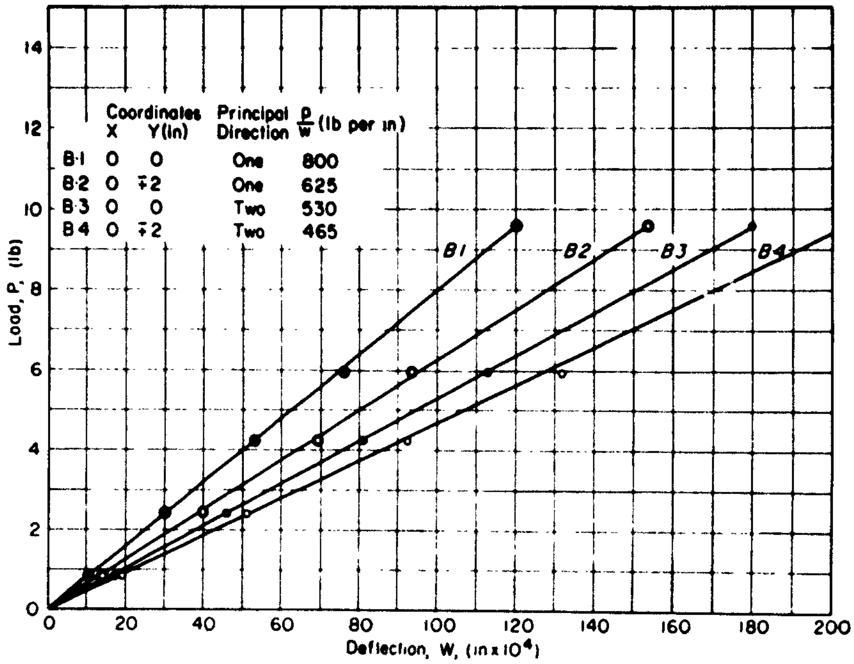

Fig. 6.—Bending Test of Paper Base Type PBG (XX).

$$(0, 2), \left[\frac{P}{W}\right] = 640 \text{ lb per in.}$$

$$a = 5.25 \text{ in.}$$
$$h = 0.116 \text{ in.}$$

The equation gives us two simultaneous equations for the elastic constants, and solving we get:

$$S_{11} = 26.5 \times 10^{-8}$$
$$S_{12} = -4.64 \times 10^{-8}$$

Therefore

$$E_x = \frac{1}{S_{11}} = \frac{1}{26.5} \times 10^8 = 3.77 \times 10^6 \text{ psi}$$

For principal direction No. 2, Fig. 7, at

$$(0,0), \left[\frac{P}{W}\right] = 603 \text{ lb per in.}$$

$$(0,2), \left[\frac{P}{W}\right] = 548 \text{ lb per in.}$$

$$a = 5.25 \text{ in.}$$
$$h = 0.116 \text{ in.}$$

The equation to be solved is

$$-ah^3\left[\frac{W_{x,y}}{P}\right] = S_{22}(7.5x^2 - 46.875) + S_{21}(7.5y^2 + 9.375x - 23.4375)$$

The equation gives us again two simultaneous equations for the elastic constants, and solving, we get:

$$S_{22} = 31.02 \times 10^{-8}$$
$$S_{21} = -4.49 \times 10^{-8}$$

Therefore

$$E_y = \frac{1}{S_{22}} = \frac{1}{31.02} \times 10^8 = 3.21 \times 10^6 \text{ psi}$$

According to the theory $S_{21} = S_{12}$. It therefore is suggested that the two somewhat different values be averaged as follows:

$$S_{12} = S_{21} = \frac{-4.64 - 4.34}{2} \times 10^{-8} = -4.49 \times 10^{-8}$$

$$S_{12} = -4.49 \times 10^{-8} = -\frac{\nu_{xy}}{E_x}$$

and

$$S_{21} = -4.49 \times 10^{-8} = -\frac{\nu_{yx}}{E_y}$$

Hence

$$\nu_{xy} = 4.49 \times 10^{-8} \times 3.77 \times 10^6 = 0.17$$
$$\nu_{yx} = 4.49 \times 10^{-8} \times 3.21 \times 10^6 = 0.15$$

Discussion and Conclusions

The method and test apparatus described in this paper appear to be quite satisfactory for the determination of the elastic constants of orthotropic materials. However, it must be emphasized that it gives elastic constants relating strain linearly to stress only for materials actually having such a characteristic over a definite stress range. It is considered that the investigation of the elastic constants of laminates by the method presented here is of considerable importance for technological uses of the material.

It appears after use of the apparatus that the method of determining deflection relative to some fixed base is preferable to the method described in the literature (1,2) using relative deflections

TABLE OF ELASTIC CONSTANTS.

Material	E_x, psi	E_y, psi	G_{xy}, psi	ν_{yx}	ν_{yx}
Glass base silicone resin	3.77×10^6	3.21×10^6	0.398×10^6	0.17	0.15
Paper base type PBG (XX)	2.65	1.99	0.665	0.31	0.23
Glass base melamine resin	5.11	4.87	1.140	0.20	0.19

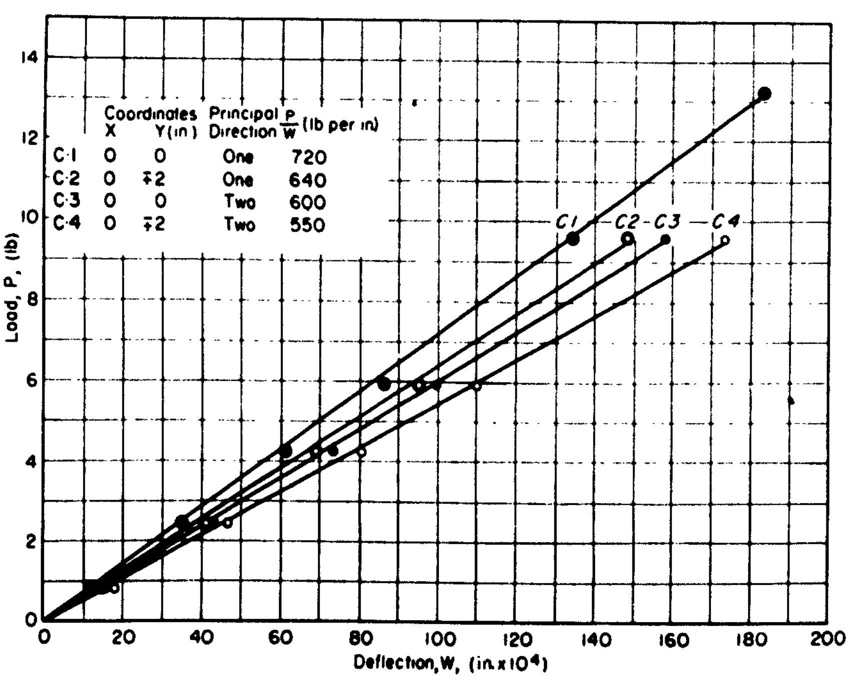

Fig. 7.—Bending Test of Glass Base Silicone Resin (GSG)

obtained by resting a measuring device completely upon the light test specimen.

The rather wide variation in bending moduli in two of the materials tested is interesting as is the fact that glass-base melamine resin is much more nearly isotropic as shown by the closeness of the E_x and E_y values. This latter material also appears quite stiff in terms of both the bending and shear moduli. It seems to be much stiffer than woods and to have stiffness of the order of some metals.

It is recommended that the method and apparatus be used in industry for studying the elastic constants of laminates and similar materials. It may be modified for investigating higher orders of anisotropy.

It is suggested that somewhat larger bending specimens be used in tests of this type so that boundary conditions may be more completely satisfied.

Acknowledgment:

The technique and test equipment described in this paper were developed in connection with an investigation being conducted at The Johns Hopkins University under contract NONR248.12 for the office of Naval Research.

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